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THE ECKERT REFERENCE FORMULATION APPLIED TO HIGH-SPEED LAMINAR BOUNDARY LAYERS OF NITROGEN AND CARBON DIOXIDE

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OF NITROGEN AND CARBON DIOXIDE

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The Eckert Reference Formulation Applied to

High-Speed Laminar Boundary Layers of Nitrogen

and Carbon Dioxide

Design calculations aimed at the determination of heat
transfer, skin friction or recovery temperature in high-speed
laminar boundary layers, can be extremely complex due to pro-
perty variations. Considerable simplification is achieved
if a constant property analysis can be used with the proper-
ties determined at suitably defined reference conditions.

In Ref. 1, Eckert presents the following expression for
the reference temperature applicable to laminar boundary
layers on surfaces at constant temperature and pressure, and
for a wide range of Mach numbers:

$$T^* = T_s + 0.50(T_w - T_s) + 0.22(T_r - T_s) \qquad (1)$$

The constants in this equation were determined such that the
reference temperature so defined, and used in a constant property
analysis, gave good agreement with all the exact calculations
available at that time. These calculations were concerned with
air and were well approximated by the above approach regardless
of the specific property variations assumed. This suggests that
the relationship assumed for the reference temperature could also
work for gases other than air, and the present work confirms this
for N_2 and CO_2.

Laminar boundary layer calculations have been carried out for N_2 and CO_2 streams allowing for variable properties (Ref. 2), thus providing the opportunity for checking the assumed reference expression on gases other than air. The property calculations are described in Ref. 3.

Ref. 1 gives tables in which the constant property results are compared with the exact calculations of Young and Janssen (4). Percentage errors are also given. To simplify comparisons the same format has been followed here for N_2 and CO_2, using the same parameters.

The tables are arranged as follows:

Table 1.   Wall at recovery temperature

        a.   Nitrogen

        b.   Carbon dioxide

Table 2.   Constant wall temperature

        a.   Nitrogen

        b.   Carbon dioxide

Table 1, column 3 and Table 2, column 4 give values of the skin friction coefficient, $(C_f \sqrt{Re})_s$ as obtained from the exact solutions. The following column in each table reports values of this same parameter $(C_f \sqrt{Re})_s$ obtained however by application of the reference temperature method (for this reason this column is labelled $(C_f \sqrt{Re})^*_s$). To obtain these values the following formulation has been used:

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$$(C_f \sqrt{Re})_s^* = (C_f \sqrt{Re})^* \sqrt{\frac{\rho^* \mu^*}{\rho_s \mu_s}} = 0.664 \sqrt{\frac{\rho^* \mu^*}{\rho_s \mu_s}} \tag{2}$$

Here we have used one of the basic reference temperature relationships, which is derived from a constant property analysis:

$$(C_f \sqrt{Re})^* = 0.664 \tag{2a}$$

The difference between the two tabulated values of $(C_f \sqrt{Re})_s$ though in a few instances larger than in Ref. 1, is still relatively small, being less than 2 or 3 per cent for both cases.

The recovery temperature is calculated from

$$T_r^* = T_s + \sqrt{P_r^*} \frac{u_s^2}{2C_p^*} \tag{3}$$

where the Prandtl number and the specific heat are evaluated at the reference temperature defined by equation (1), it is compared in columns 6 and 7 of Table 1 with the value of the recovery temperature T_r found from the boundary layer analysis. The percentage errors are no larger than those in Ref. 1.

In applying the reference temperature method to the calculation of the heat transfer coefficient the analogy expression relating heat transfer and skin friction is utilized:

$$\left(\frac{C_f}{2 \, St}\right)^* = (P_r^*)^{2/3} \tag{4}$$

To compare the boundary layer results with the reference temperature formula of eq. (4) the value of $(C_f/2 \, St)^*$ is determined from the exact calculations as follows:

3

$$\left(\frac{C_f}{2\ St}\right)^* = \left(\frac{C_f}{2\ St}\right)_s \frac{C_p^*}{C_{p_s}} \tag{5}$$

The values obtained from equations (5) and (4) are tabulated in columns (8) and (9) respectively of Table 2. The percentage error is less than 10 percent in all cases except one, with the deviations in general being higher for CO_2 than for N_2. This agreement is as good as that reported in Reference 1, and in the case of Nitrogen it is somewhat better.

The alternative procedure proposed by Eckert is to define the heat transfer coefficient in terms of enthalpy difference:

$$q_w = h_i\ (i_r - i_w) \tag{6}$$

where now the enthalpy recovery factor r_i is given by

$$r_i = \frac{i_r - i_s}{u_s^2/2} \tag{7}$$

The value of the enthalpy recovery factor r_i as taken from the boundary layer solutions is reported in column 9 of Table 2 and compared with the approximate value given as $\sqrt{P_r^*}$ in column 10. The resulting error in the use of the reference method is comparable to that in Reference 1, being of the order of 1 to 2 percent.

The heat transfer coefficient h_i is reported in dimensionless form as $(C_f/2\ St_i)$ and the boundary layer calculations are shown in column 10 of Table 2. The reference method

proposed a value of $(P_r^*)^{2/3}$ for this ratio. The agreement of the approximate and exact values is even better than that reported in Reference 1.

Conclusions and Recommendation

The use of the reference temperature method yields approximations for N_2 and CO_2 laminar boundary layers which are as good as those of Ref. 1 for air. If the specific heat varies over a wide range or if dissociation occurs it is recommended as in Reference 1, though this remains to be verified, that properties be evaluated at a reference enthalpy rather than the reference temperature, where the reference enthalpy is given by:

$$i^* = i_s + 0.5(i_w - i_s) + 0.22(i_r - i_s)$$

Nomenclature

$C_f = \dfrac{\tau_w}{\frac{1}{2} \rho u_s^2}$ = local skin friction coefficient

C_p = Specific heat at constant pressure

$h = q_w/(T_r - T_w)$ = local heat transfer coefficient

$h_i = q_w/(i_r - i_w)$ = local enthalpy difference heat transfer coefficient

i = enthalpy

Pr = Prandtl number

q = heat flow per unit time and area

$r = (T_r - T_s)/\dfrac{u_s^2}{2C_p}$ = recovery factor

$r_i = (i_r - i_s)/\dfrac{u_s^2}{2}$ = enthalpy recovery factor

$St_i = h_i/\rho u_s$ = Stanton number based on enthalpy difference

T = temperature

u = velocity

μ = viscosity

ρ = density

τ = shearing stress

Subscripts

i = based on enthalpy

r = recovery

s = in free stream

w = at wall

Superscripts

$*$ = reference condition

TABLE 1 a

N_2 Recovery Case

$T_s \, °K$	M_s	$(C_f \sqrt{Re})_s$	$(C_f \sqrt{Re})_s^*$	per cent	$T_r \, °K$	$T_r^* \, °K$	per cent	r_i	$\sqrt{P_r^*}$	per cent
218	4	0.574	0.569	-0.9	791	790	-0.1	0.836	0.846	1.2
555	4	0.564	0.560	-0.7	1843	1812	-1.7	0.839	0.852	1.5
1110	4	0.574	0.568	-1.0	3409	3382	-0.8	0.841	0.851	1.2
218	8	0.485	0.481	-0.8	2274	2199	-3.3	0.835	0.853	2.2
218	12	0.432	0.426	-1.4	4570	4438	-2.9	0.835	0.852	2.0

TABLE 1 b

CO_2 Recovery Case

$T_s \, °K$	M_s	$(C_f \sqrt{Re})_s$	$(C_f \sqrt{Re})_s^*$	per cent	$T_r \, °K$	$T_r^* \, °K$	per cent	r_i	$\sqrt{P_r^*}$	per cent
218	4	0.640	0.640	0	634	610	-3.8	0.854	0.860	0.7
555	4	0.612	0.610	-0.3	1271	1248	-1.8	0.834	0.842	1.0
1110	4	0.607	0.604	-0.5	2329	2317	-0.5	0.827	0.836	1.1
218	8	0.575	0.578	0.5	1554	1452	-6.6	0.832	0.841	1.1
555	8	0.536	0.534	-0.4	3103	3020	-2.7	0.821	0.834	1.6
218	12	0.521	0.522	0.2	2917	2752	-5.7	0.821	0.836	1.8

TABLE 2 a

N_2 Constant Wall Temperature

$T_s \, {}^{\circ}K$	$T_w \, {}^{\circ}K$	M_s	$(C_f \sqrt{Re})_s$	$(C_f \sqrt{Re})^*_s$	per cent	$(C_f/2St)^*$	$(Pr^*)^{2/3}$	per cent	$(C_f/2St_i)$	per cent
218	436	0	0.633	0.612	-3.3	0.806	0.804	-0.2	0.807	0.1
218	1308	0	0.553	0.547	-1.0	0.798	0.801	0.4	0.801	0.4
1110	555	0	0.707	0.693	-2.0	0.811	0.801	-1.2	0.810	-0.1
218	436	4	0.607	0.597	-1.6	0.778	0.800	2.8	0.800	2.8
218	1308	4	0.540	0.529	-2.0	0.776	0.799	3.0	0.799	3.0
555	555	4	0.624	0.615	-1.4	0.768	0.800	4.2	0.806	4.9
555	1110	4	0.592	0.582	-1.7	0.776	0.802	3.4	0.807	4.0
555	1665	4	0.571	0.559	-2.1	0.782	0.806	3.1	0.805	2.9
1110	555	4	0.651	0.638	-2.0	0.791	0.806	1.9	0.809	2.3
218	436	8	0.555	0.542	-2.3	0.749	0.801	6.9	0.802	7.1
218	1308	8	0.513	0.503	-1.9	0.764	0.803	5.1	0.802	5.0
218	436	12	0.509	0.493	-3.1	0.771	0.805	4.4	0.803	4.2
218	1308	12	0.485	0.473	-2.5	0.776	0.806	3.9	0.803	3.5

TABLE 2 b

CO_2 Constant Wall Temperature

$T_s\,{}^{\circ}K$	$T_w\,{}^{\circ}K$	M_s	$(C_f\sqrt{Re})_s$	$(C_f\sqrt{Re})^*_s$	per cent	$(C_f/2St)^*$	$(Pr^*)^{2/3}$	per cent	$(C_f/2St_i)$	per cent
218	436	0	0.661	0.633	-1.2	0.838	0.834	-0.5	0.835	-0.4
218	1308	0	0.608	0.606	-0.33	0.798	0.808	1.3	0.802	0.5
1110	555	0	0.697	0.683	-2.0	0.818	0.801	-2.1	0.813	-0.6
218	436	4	0.652	0.636	-2.5	0.759	0.825	8.7	0.813	7.1
218	1308	4	0.600	0.593	-1.2	0.775	0.800	3.2	0.794	2.5
555	555	4	0.648	0.635	-2.0	0.762	0.805	5.6	0.802	5.2
555	1110	4	0.619	0.607	-1.9	0.757	0.797	5.3	0.786	3.8
555	1665	4	0.595	0.592	-0.5	0.773	0.792	2.5	0.791	2.3
1110	555	4	0.669	0.658	-1.6	0.783	0.793	1.3	0.799	2.0
218	436	8	0.627	0.627	0	0.726	0.809	11.4	0.797	9.8
218	1308	8	0.585	0.575	-1.7	0.736	0.796	8.2	0.778	5.7
555	555	8	0.608	0.601	-1.2	0.757	0.793	4.8	0.789	4.2
555	1110	8	0.589	0.581	-1.4	0.752	0.790	5.1	0.782	4.0
555	1665	8	0.571	0.563	-1.4	0.754	0.789	4.6	0.778	3.2
218	436	12	0.595	0.588	-1.2	0.742	0.797	7.4	0.790	6.5
218	1308	12	0.564	0.560	-0.7	0.743	0.791	6.5	0.779	4.8

References

1. "Engineering Relations for Heat Transfer and Friction
   in High-Velocity Laminar and Turbulent Boundary-Layer
   Flow over Surfaces with Constant Pressure and Temperature,"
   E. R. G. Eckert, Trans. A.S.M.E., Vol. 78, August, 1956,
   p. 1273.

2. "Mass Transfer Cooling In Laminar Boundary Layers with
   Hydrogen Injected into Nitrogen and Carbon Dioxide Streams,"
   C. S. Liu, M.M.E. Thesis, University of Delaware, June, 1965.

3. "Properties of Hydrogen:Nitrogen and Hydrogen:Carbon
   Dioxide Mixtures," H.A. Simon, C.S. Liu and J. P. Hartnett,
   University of Delaware, Department of Mechanical Engineering
   T. N. 3, January, 1965.

4. "The Compressible Boundary Layer," G. B. W. Young and
   E. Janssen, Jnl. of the Aero. Sciences, Vol. 19, 1952.

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